HAMWIC TRUST

Talbot Primary School
Progression in Calculation
Reception – Year 6
2020

Introduction

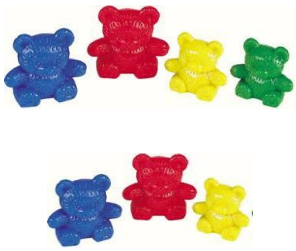
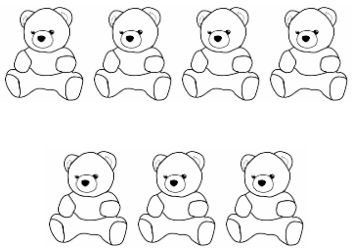
The aim of this document is to show how each of the four operations (addition, subtraction, multiplication, division) are taught at Talbot. It can be used to support any learning you may choose to focus on with your child at home and a paper copy can be requested from the office.

Strategies are introduced at three different stages (The Early Years Foundation Stage, Key Stage 1 and Key Stage 2) however, children will continue to use and extend strategies that they learnt at a younger age as they become older and start working with larger numbers.

Concrete, Pictorial, Abstract

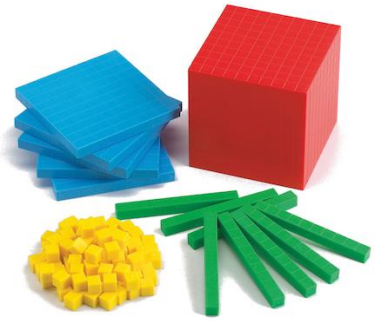
At Talbot, we follow the 'concrete, pictorial, abstract' model when introducing a new mathematical concept. This allows children to practise the new skill with physical resources, before moving on to representing it visually and then with mathematical symbols.

Example: I have four teddies and you have three. How many do we have altogether?

Concrete	Pictorial	Abstract
Children count the concrete resources.	Children draw and count the pictures.	Children represent the problem formally.
		$4 + 3 = 7$

This example is a simple one, but the same principles can be applied to much more advanced mathematical concepts.

Although we use a wide range of concrete resources at Talbot, we have found that counting apparatus, Dienes blocks and place value counters are particularly effective for teaching calculation. Some key visual aids that we use are the number line and the hundred square.

Counting Apparatus	Dienes Blocks	Place Value Counters
		

Fluency, Reasoning and Problem Solving

The 2014 National Curriculum specifies that all children should be **fluent** in the fundamentals of mathematics, be able to **reason** mathematically and be able to **solve problems** by applying their knowledge. In mathematics lessons at Talbot, fluency, reasoning and problem solving are all developed alongside each other. This calculation policy focuses on developing children's fluency.

Fluency can be defined as the ability to recall and apply knowledge rapidly and accurately, which is underpinned by conceptual understanding. This is supported by the confident use of mental methods. Children at Talbot are taught a variety of mental methods, some supported by jottings, before moving on to learning the formal written methods.

When approaching a calculation question, children should:

- First ask themselves whether a mental method would be appropriate.
- Use estimation to judge whether their answers are reasonable.
- Check their answers using an appropriate strategy.

Quick Recall

It is extremely important that children memorise certain 'number facts' and can recall these without hesitation. This quick recall supports all other areas of mathematics, as it allows children to work flexibly when presented with unfamiliar problems and reduces cognitive load.

We strongly encourage the use of the NumBots and TTRockstars applications at home to help children memorise these facts in a fun and engaging way.

The number facts are taught in this order:

- Number bonds to 10
- Addition and subtraction facts within 10
- Number bonds to 20
- Addition and subtraction facts within 20
- Doubles and halves within 20
- 2, 5 and 10 times tables
- Know what must be added to any two-digit number to make 100
- 3, 4 and 8 times tables
- 6, 7, 9, 11 and 12 times tables
- Prime numbers to 20
- Square numbers up to 12^2
- Cube numbers up to 5^3

Counting and Early Number

In the Early Years Foundation Stage (Reception) children are taught key number concepts that support later calculation.

The one-one principle

Children match one 'counting word' with each object in a set to be counted. They say one number word, and only one number word, for each object being counted. Children often use touch-counting and may want to move objects from one pile to another as they count.

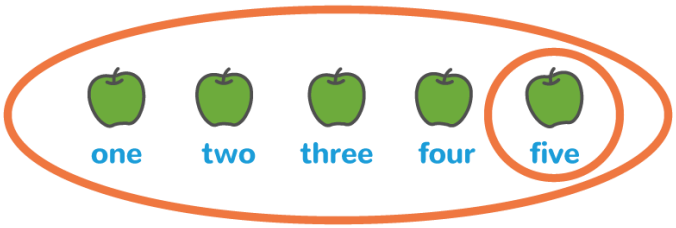
The stable-order principle

Children learn to say the counting words in the same order every time. Children learn the order of numbers through songs, rhymes and stories.

One, two, three, four five,
Once I caught a fish alive,
Six, seven, eight, nine, ten,
Then I let it go again.

The cardinal principle

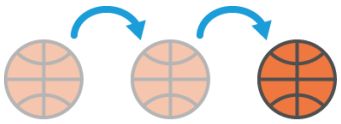
Children understand that the purpose of counting is to find out how many items there are altogether. They understand that the last number word that they say is not just a name given to the last item in a set, but that it represents the total number of items.



The abstract principle

Children understand that counting can be applied to any set, not just physical objects. They start to count items that they can't see or touch, but can hear or imagine, for example counting hops or claps.

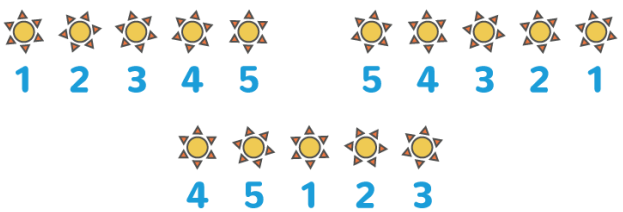
1, 2 beats of the drum



1, 2 bounces of the ball

The order-irrelevance principle

Children understand that the total number of items in a set does not change if you count them in a different order. It doesn't matter which item you start on, the total stays the same.



Subitising

Children recognise the number of objects in a set without using counting. Children start to recognise numbers of fingers, numbers on a dice, or Numicon without needing to actually count.

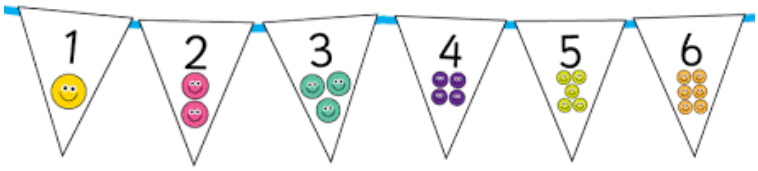
When these principles are secure, children start to perform early calculations such as:

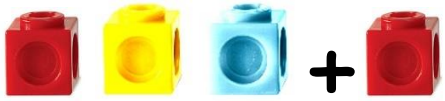
- Counting more than one set of objects
- Taking a number of objects out of a set
- Finding one more and one less than a given number
- Counting in twos

Progression in Addition

Introduced in the Early Years Foundation Stage

Count on in steps of one and find one more than a given number

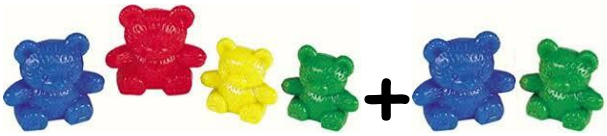




'One more than three is four'

Count all

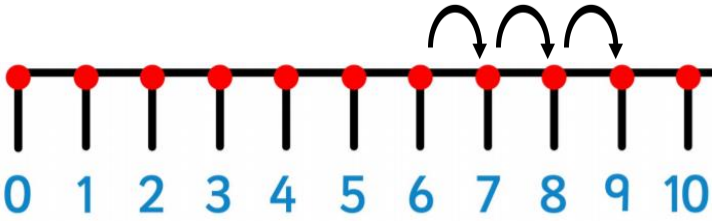
Combine two groups of objects and recount them



'Four add two is six'

Count on

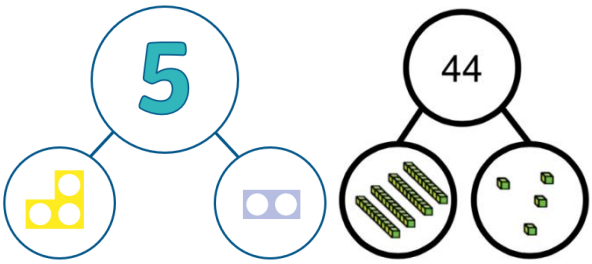
Start at a number and count on in ones



'Six add three is nine'

Children should learn that it is more efficient to count on from the larger number.

Part-whole

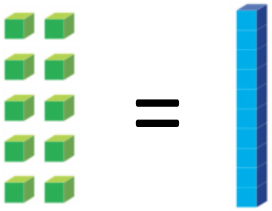


This model should be used to develop addition and subtraction alongside each other, so that children begin to see the inverse relationship.

Children should begin to understand that addition can be done in any order.

Introduced in Key Stage 1

Regrouping to make ten

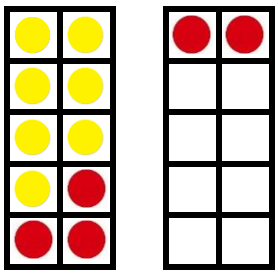


'Ten ones is equal to one ten'

This is a key concept that supports formal column addition later on.

Make ten or Bridging

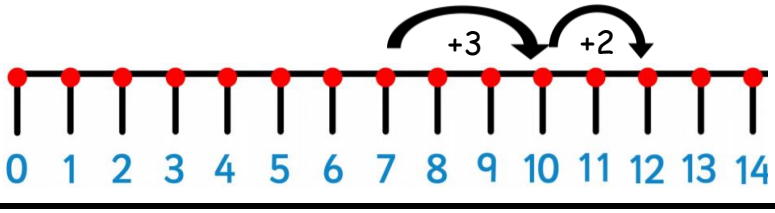
Partition the smaller number to bridge through the nearest ten.



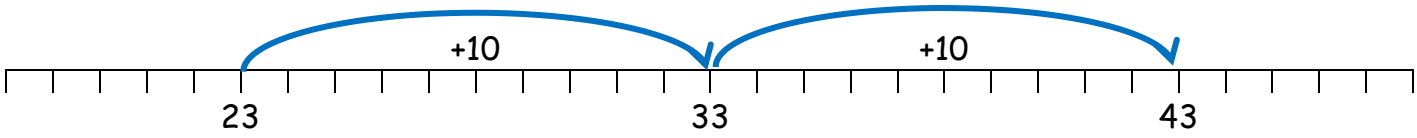
7 + 5 = 12

/\

3 2



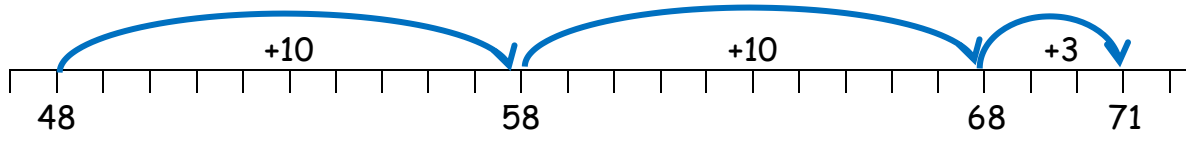
Count on in tens from any number



In later years, this is extended to counting on in hundreds or thousands from any number, and for counting on in steps of 0.1 for decimal numbers.

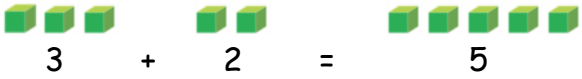
Partition the smaller number to count on in tens and ones

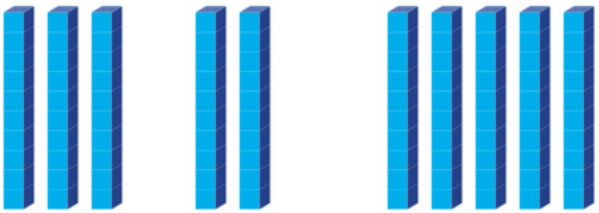
48 + 23 = 71
/\
20 3



*This strategy can be combined with the **bridging** strategy to add 3 in steps of 2 and 1.*
In later years, this strategy is extended to partitioning 3-digit numbers.

Use known facts to deduce others



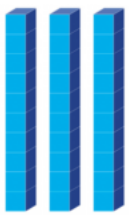
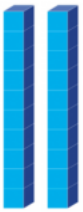


'If three add two equals five, then thirty add twenty equals fifty'

In later years this strategy is extended to use with hundreds and thousands.

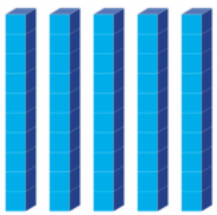
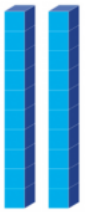
Partition to add (no exchanging)

32+27

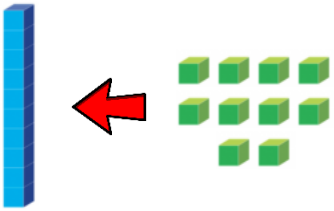
Tens	Ones
	
	
30 + 20 = 50	2 + 7 = 9
Answer: 59	

Partition to add (with exchanging)

57 + 28

Tens	Ones
	
	
50 + 20 = 70	7 + 8 = 15
Answer: 85	

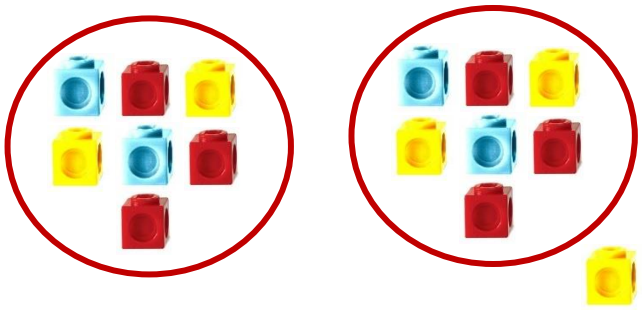
The key learning point is that ten ones have been exchanged for one ten, leaving five ones in the ones column.



Introduced in Key Stage 2

Near doubles

When children are confident with doubling, this can be used for quick mental calculation.



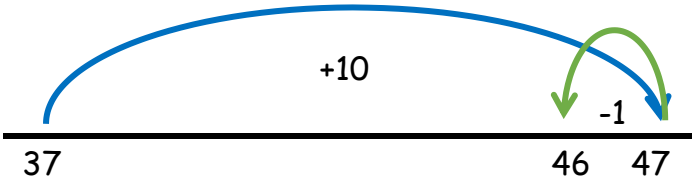
7 8

'7 add 8 is the same as double 7 and 1 more'

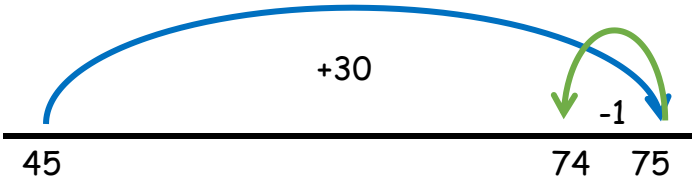
In later years, this is extended to use with larger numbers and with decimals.
150 + 160 = double 150 + 10
2.5 + 2.6 = double 2.5 + 0.1

Compensation

When adding a number that is close to a multiple of ten, children add the multiple of ten and then adjust the answer accordingly.



'To add 9 we can add 10 then subtract 1'



'45 + 29 is the same as 45 + 30, take away 1'
In later years, this is extended to numbers close to multiples of 100 (e.g. 99).

Column method without exchanging

	Th	H	T	O
	3	6	3	7
+		1	4	2
	3	7	7	9

Column method with exchanging

	Th	H	T	O
	4	6	1	3
+		8	2	9
	5	4	4	2
	1		1	

Column method with decimals

	T	O	•	$\frac{1}{10}$	$\frac{1}{100}$
	1	8	•	2	9
+		3	•	5	6
	2	1	•	8	5
	1			1	

Key Vocabulary for Addition

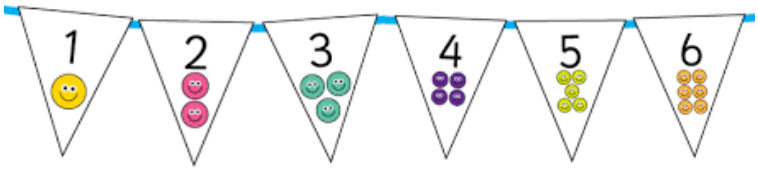
For combining two groups
How much altogether?
How many altogether?
Total
Sum

For counting on from a starting point
Start at and count on
Increase by
Go up by

Progression in Subtraction

Introduced in the Early Years Foundation Stage

Count back in steps of one and find one less than a given number



'One less than four is three'

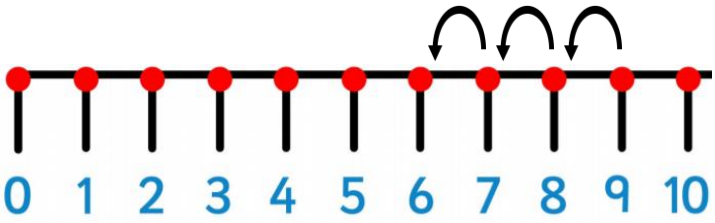
Take away

Take objects away from a group and count the remaining objects

'Six take away two is four'

Count back

Start at a number and count back in ones

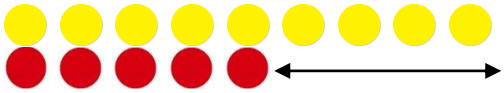


'Nine take away three is six'

Introduced in Key Stage 1

Find the difference

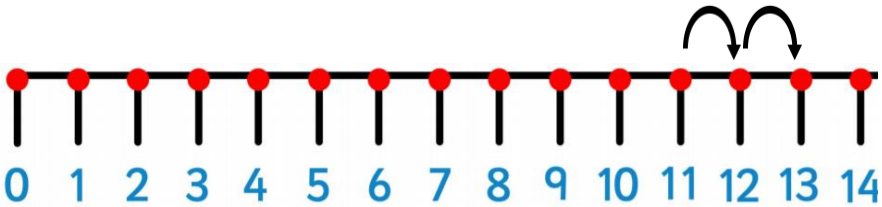
Used to compare two groups of objects.



'The difference between nine and five is four'

Understanding subtraction as difference is a key concept which supports calculation with negative numbers later on.

Count on to find the difference

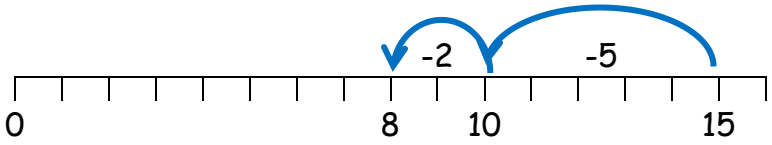


'The difference between 11 and 13 is two'

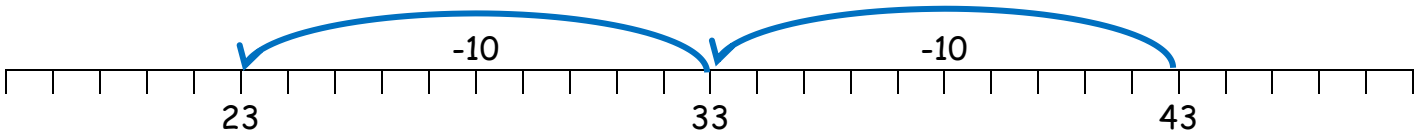
Children should start to realise that counting on is more efficient than counting back when finding the difference between two numbers that are close together.

Subtract single-digit numbers from two-digit numbers, often bridging through 10

15 – 7 = 8
/\
5 2



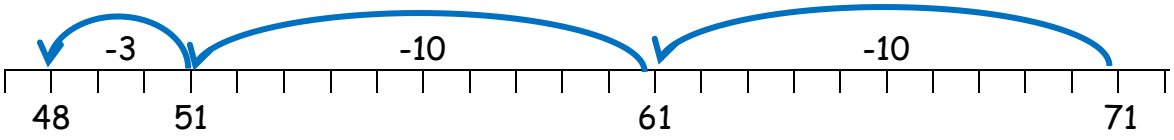
Count back in tens from any number



In later years, this is extended to counting back in hundreds or thousands from any number, and for counting back in steps of 0.1 for decimal numbers.

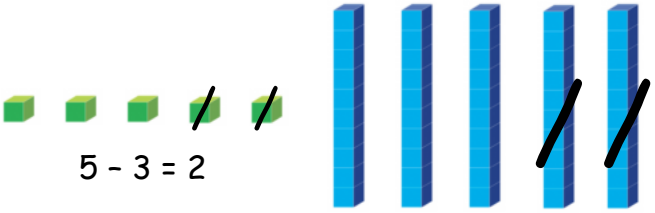
Partition the number being subtracted to count back in tens and ones

71 - 23 = 48
/\
20 3



*This strategy can be combined with the **bridging** strategy to subtract 3 in steps of 1 and 2. In later years, this strategy is extended to partitioning 3-digit numbers.*

Use known facts to deduce others



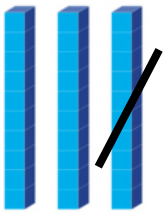
5 – 3 = 2

50 – 30 = 20

'If five take away three equals two, then fifty take away thirty equals twenty'.
In later years this strategy is extended to use with hundreds and thousands.

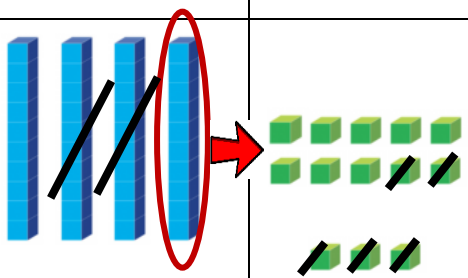
Partition to subtract (no exchanging)

38 - 13

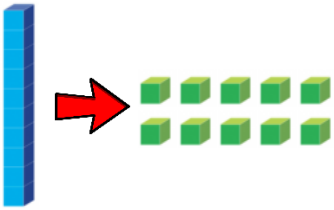
Tens	Ones
	
30 – 10 = 20	8 – 3 = 5
Answer: 25	

Partition to subtract (with exchanging)

43 - 25

Tens	Ones
	
One ten has been used as ten ones 30 – 20 = 10	Can't do 3 – 5 13 – 5 = 8
Answer: 18	

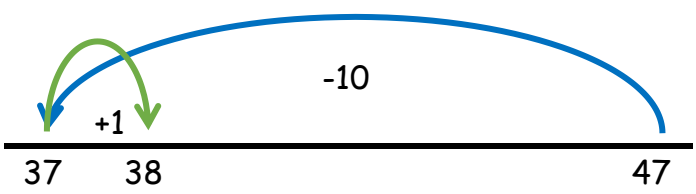
The key learning point is that because you cannot subtract 5 from 3, you must use one of the tens and exchange it for ten ones.



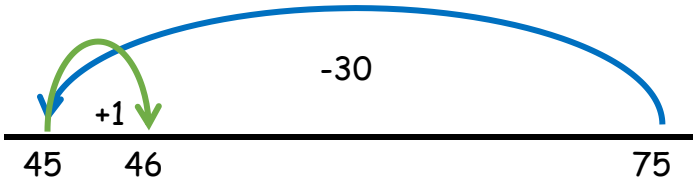
Introduced in Key Stage 2

Compensation

When subtracting a number that is close to a multiple of ten, children subtract the multiple of ten and then adjust the answer accordingly.



'To subtract 9 we can subtract 10 then add 1'



'75 - 29 is the same as 75 - 30, add 1'

In later years, this is extended to numbers close to multiples of 100 (e.g. 99).

Column method without exchanging

	Th	H	T	O
	3	6	3	7
-	2	4	2	3
	1	2	1	4

Column method with exchanging

	Th	H	T	O
	4	$^{3}\cancel{4}$	$^{1}1$	3
-	1	3	5	2
	3	0	6	1

Column method with exchanging across two columns

	H	T	O
	$^{3}\cancel{4}$	$^{9\,1}\cancel{0}$	$^{1}3$
-	1	3	7
	2	6	6

Column method with decimals

	O	•	$\frac{1}{10}$	$\frac{1}{100}$
	8	•	5	9
-	3	•	2	6
	5	•	3	3

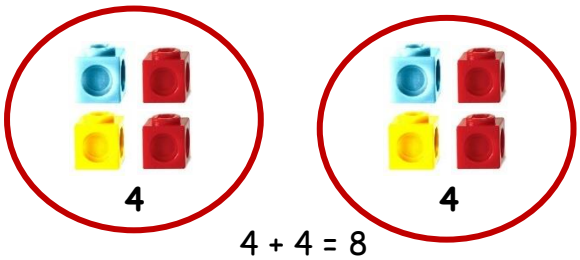
Key Vocabulary for Subtraction

For taking away from a group	For counting back from a starting point	For finding the difference
Take away	Start at and count back	Difference
How much left?	Reduce	How many more?
How many left?	Decrease	How many fewer?
How many are not…?	Go down	How much bigger?
		How much smaller?

Progression in Multiplication

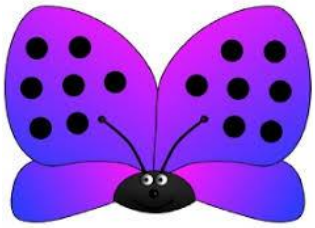
Introduced in the Early Years Foundation Stage

Doubling



4 + 4 = 8

'Double four is eight'



'Double 7 is 14'

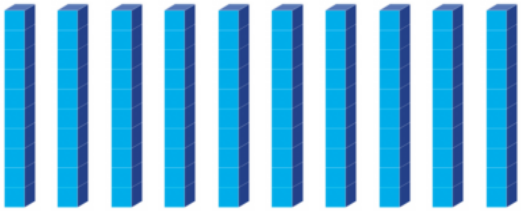
Children initially use counting to double, before memorising doubles up to 20.
In later years, children learn to double any two-digit number using partitioning.

Introduced in Key Stage 1

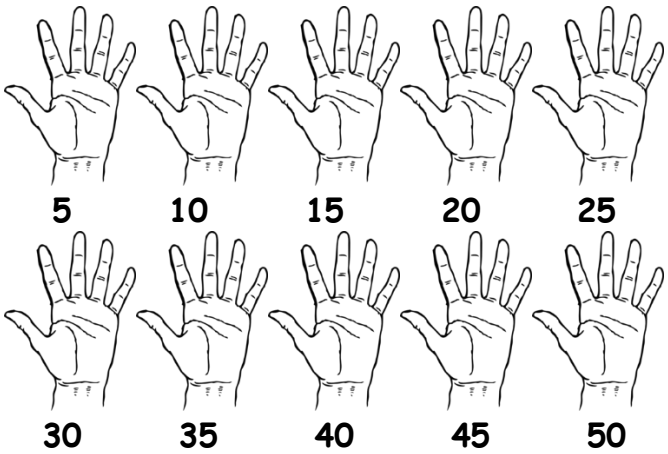
Count in steps of 2, 5 and 10

'Two, four, six, eight, ten'

1	2	3	4	5	6	7	8	9	10



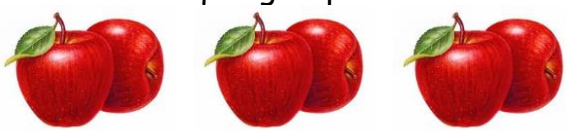
10, 20, 30, 40, 50, 60, 70, 80, 90, 100



Children learn that numbers that are divisible by 2 are even, and other numbers are odd.
In later years, children learn to count in steps of all numbers up to 12.

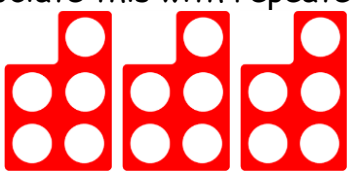
Repeated addition

Children make equal groups and count the total. They associate this with repeated addition.



2 + 2 + 2 = 6

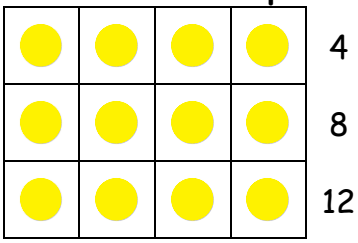
'3 groups of 2 equals 6'



5 + 5 + 5 = 15

'3 groups of 5 equals 15'

Represent multiplication as an array

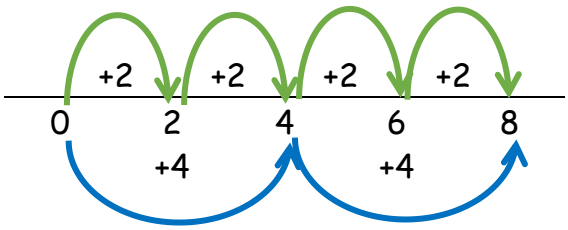


'3 × 4 = 12'

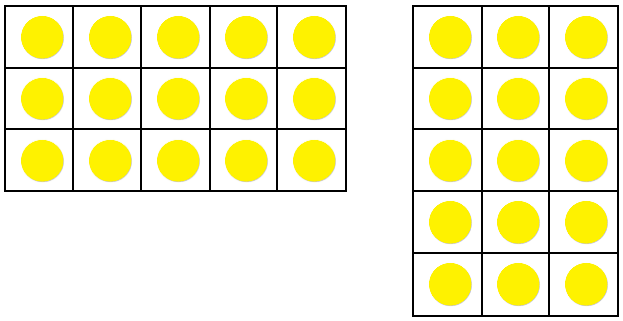
Children use step counting rather than counting each item individually.

Multiplication can be done in any order

'2 jumps of 4 is the same as 4 jumps of 2'



'3 rows of 5 is the same as 5 rows of 3'



Use known facts to deduce others

'If 3 × 5 = 15 then 3 × 50 = 150'

5	5	5

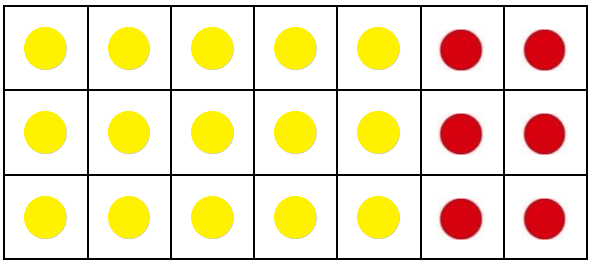
50	50	50

Introduced in Key Stage 2

Numbers can be partitioned, multiplied, then recombined

Children partition numbers they are familiar with and see that the answer stays the same.

3 × 5 = 15 3 × 2 = 6



15 + 6 = 21

3 × 7 = 21

/\

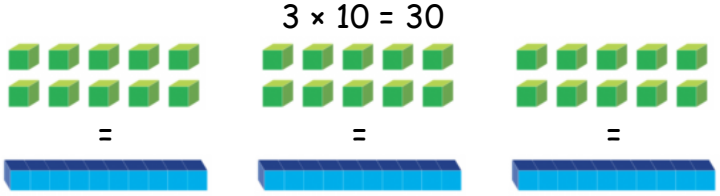
5 2

This is a key concept that children need in order to understand formal column multiplication.

Multiply by 10

Children learn that when multiplying by 10, the place value of each digit increases.

3 × 10 = 30



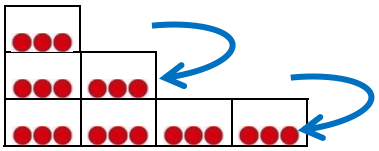
Hundreds	Tens	Ones
	2	3
2	3	0

23 × 10 = 30

It is important that children do not associate this with 'adding a zero' as this will not work when they start using decimal numbers.

Multiply by 4 by doubling and doubling again

3 × 4 = 12



'Double 3 is 6. Double 6 is 12.'

Factorisation

Children learn that they can split numbers into factors and the answer remains the same.

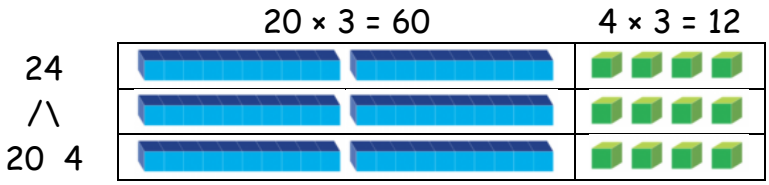
3 × 22 = 3 × 11 × 2 = 66 → 33

3 × 40 = 3 × 4 × 10 = 120 → 12

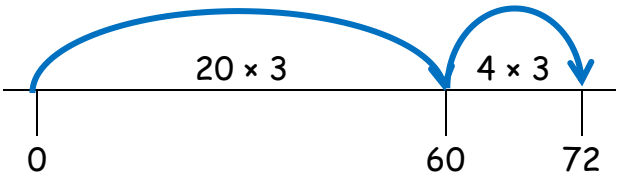
Children use their knowledge of halving and doubling to make calculations easier.

Partition to multiply a 2-digit number by a 1-digit number

24 × 3



24 × 3 = 60 + 12 = 72



Multiply by a 1-digit number

	Th	H	T	O
		6	3	7
×				3
	1	9	1	1
		1	2	

Multiply by a 2-digit number

	Th	H	T	O	
		2	3	7	
×			2	3	
		7	1	1	(3 × 237)
		1	2		
+	4	7	4	0	(20 × 237)
		1			
	5	4	5	1	
	1				

This strategy relies heavily on children's understanding that numbers can be partitioned, multiplied and recombined, alongside the effects of multiplying by 10.

Multiply decimals by a 1-digit number

	O	•	$\frac{1}{10}$	$\frac{1}{100}$
	1	•	2	9
×				4
	5	•	1	6
	1		3	

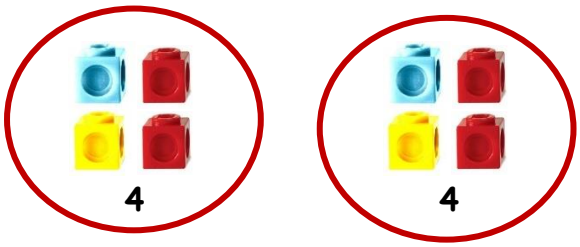
Key Vocabulary for Multiplication

For repeated addition	For Scaling
Sets of	Scaling
Lots of	Doubling
Altogether	Trebling
Per	So many times bigger/longer/heavier than…
Each	So many times as much as…
Product	

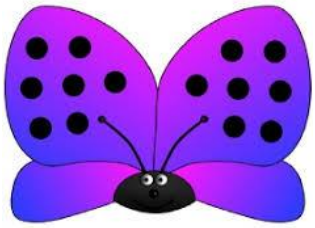
Progression in Division

Introduced in the Early Years Foundation Stage

Halving



'Half of eight is four'



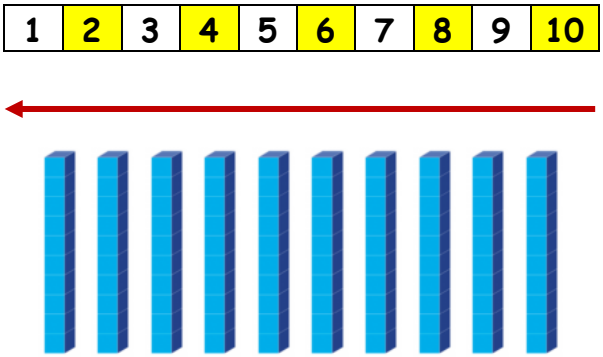
'Half of 14 is 7'

Children initially use counting to halve, before memorising halves up to 20.

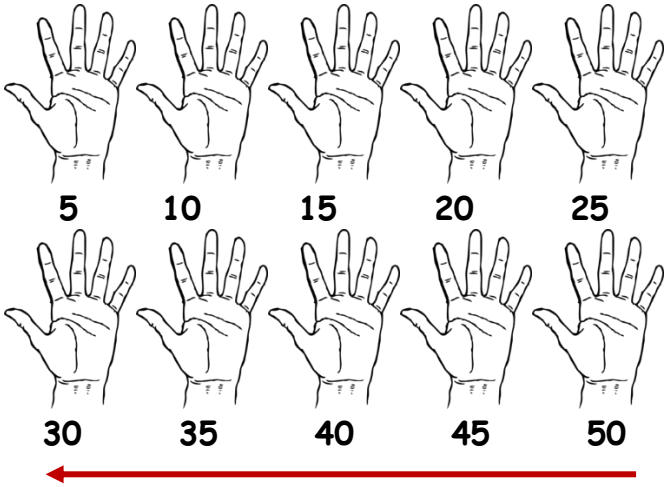
Introduced in Key Stage 1

Counting back in steps of 2, 5 and 10

'Ten, eight, six, four, two, zero'



10, 20, 30, 40, 50, 60, 70, 80, 90, 100



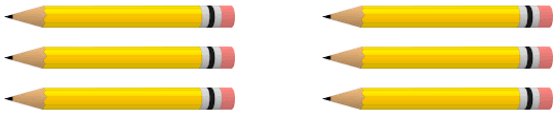
Fair sharing

'If I share six apples fairly between three people, everyone gets two apples each.'

6 ÷ 3 = 2

Grouping

'If I put 6 pencils into groups of 3, I can make 2 groups.'

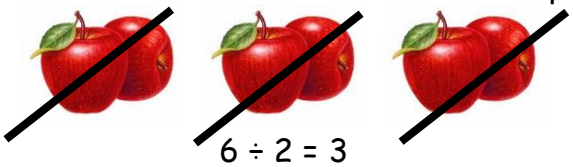


6 ÷ 3 = 2

Children link this to repeated subtraction.
This is essential for understanding the formal methods of short and long division.

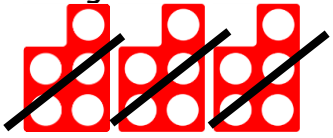
Repeated subtraction

Children subtract in equal steps from a starting number.



6 ÷ 2 = 3

'I can subtract 2 from 6 three times'

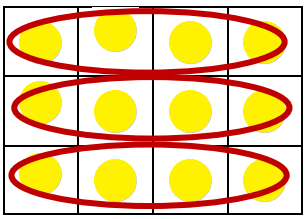


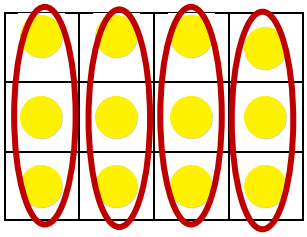
15 ÷ 5 = 3

'I can subtract 5 from 15 three times'

Children learn that they can also count up to a target number from 0.

Represent division as an array





12 can be made from
3 groups of 4,
or 4 groups of 3.

Ratio and inverse of multiplication

Amy has twice as many teddies as Bob.
Amy has 8 teddies.
How many does Bob have?

Children link the language of 'so many times less/fewer than' with division.

Use known facts to deduce others

'If 15 ÷ 3 = 5 then 150 ÷ 3 = 50'

5	5	5
50	50	50

Introduced in Key Stage 2

Divide by 10

Children learn that when dividing by 10, the place value of each digit decreases.

30 ÷ 10 = 3
'I can make 3 groups of 10'

Hundreds	Tens	Ones
2	3	0
	2	3

230 ÷ 10 = 23

It is important that children do not associate this with 'removing a zero' as this will not work when they start using decimal numbers.

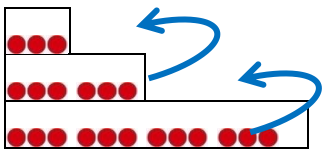
Partition to halve

Half of 64 = 32
/\
60 4

Half of 60 = 30
Half of 4 = 2

Divide by 4 by halving and halving again

12 ÷ 4 = 3



Half of 12 is 6. Half of 6 is 3.

Partition to divide

936 ÷ 3 = 312

9 3 6
/ | \
900 30 6

900 ÷ 3 = 300
30 ÷ 3 = 10
6 ÷ 3 = 2

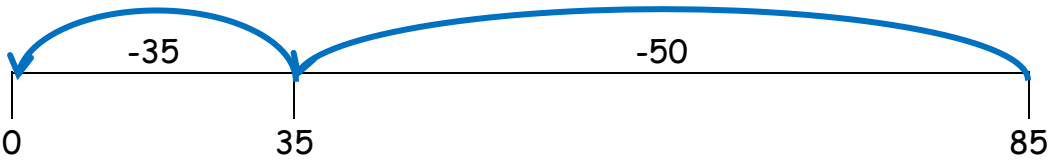
Factorisation

144 ÷ 24
/\
12 × 2
144 ÷ 12 = 12
12 ÷ 2 = 6

'I know that 2 and 12 are a factor pair of 24 so I can divide by 12 and then by 2'

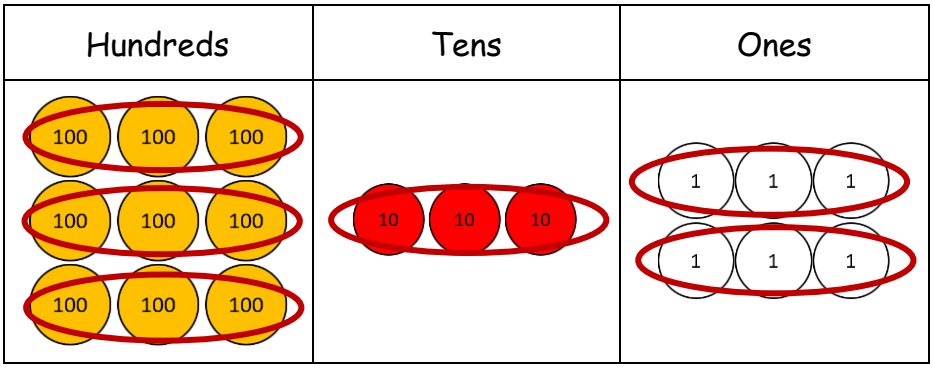
Partition for more efficient repeated subtraction

85 ÷ 5 = 17
/\
50 35



Children subtract familiar 'chunks' from the starting number until they reach zero.

Short division supported by place value equipment



936 ÷ 3

Children count the groups of 3 in each column.

	H	T	O
		3	1 2
3	9	3	6

Children start with numbers which can be grouped with no remainders.
When they progress to numbers where there is a ten 'left over' they learn to exchange this for ten ones, which can then be grouped.

Short division

	1	3	4
7	9	$^{2}3$	$^{2}8$

Short division with remainders

	1	3	4	r 1
7	9	$^{2}3$	$^{2}9$	

Short division with remainders as decimals

	1	5	8 •	2	
5	7	$^{2}9$	$^{4}1$ •	$^{1}0$	

Long division

		2	8	r 12
15	4	3	2	
-	3	0	↓	(15 × 2)
	1	3	2	
-	1	2	0	(15 × 8)
		1	2	

Key Vocabulary for Division

For Sharing	For Grouping	For Ratio
Shared equally between How much/many each?	How many groups? How many sets?	How many times greater/longer/heavier?

National Curriculum Statements for Addition and Subtraction						
EYFS	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.	Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs. Represent and use number bonds and related subtraction facts within 20. Add and subtract one-digit and two-digit numbers to 20, including zero. Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.	Solve problems with addition and subtraction: - using concrete objects and pictorial representations, including those involving numbers, quantities and measures - applying their increasing knowledge of mental and written methods. Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers. Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	Add and subtract numbers mentally, including: - a three-digit number and ones - a three-digit number and tens - a three-digit number and hundreds. Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Estimate the answer to a calculation and use inverse operations to check answers. Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate. Estimate and use inverse operations to check answers to a calculation Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Add and subtract numbers mentally with increasingly large numbers. Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy. Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	Perform mental calculations, including with mixed operations and large numbers. Use their knowledge of the order of operations to carry out calculations involving the four operations. Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Solve problems involving addition, subtraction, multiplication and division. Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

National Curriculum Statements for Multiplication and Division						
EYFS	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
They solve problems, including doubling, halving and sharing.	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs. Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	Recall multiplication and division facts for multiplication tables up to 12 × 12. Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers. Recognise and use factor pairs and commutativity in mental calculations. Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers. Establish whether a number up to 100 is prime and recall prime numbers up to 19. Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Multiply and divide numbers mentally drawing upon known facts. Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}). Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes. Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. Perform mental calculations, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers. Use their knowledge of the order of operations to carry out calculations involving the four operations. Solve problems involving addition, subtraction, multiplication and division. Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.